# Applications of Stochastic Hybrid Systems in Portfolio Optimization

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**Abstract**

Stochastic optimal control as an important part of financial mathematics is widely used in various applications. We present stochastic hybrid models to account for regime switching dynamics. Abrupt fluctuations that are usual part of financial markets have been captured in stochastic models with a motion part and additionally a jump part. The semi-martingale dynamic/sudden change is governed by a Markov process. These models allow for more realistic investigation of portfolio optimization and utility maximization in financial markets. The solution of these models is provided through specifications of Hamilton-Jacobi-Bellman functions, where the price process $X(t)$ switches from one jump-diffusion path to another as the discrete component, and $\theta(t)$ moves from one step to another, with an initial conditions of $X(0) = X_0$, $\theta(0) = \theta_0$.

In addition to Stochastic Hybrid Systems (SHS), filtration in the models accounts for the efficient hypothesis methodology. Market efficiency suggests that competitive prices must reflect all price changes that perform a random walk with no predictable bias, and is defined as available information in relationship to returns, with the incorporated insider information in models. Filtration gives the necessary tool to develop optimal consumption and investment rates, where an insider investor gains larger information by modeling a general filtration $(G_t)$, than the one available to a honest investor.

Furthermore, explicit consideration of risk aversion in an optimal investment and consumption problem allows for optimality conditions that are related to specific risk types in a market. In extension, approximations for the non-linear Hamilton-Jacobi Partial Differential Equations (PDE) with boundary conditions can be applied as solutions to Hybrid Stochastic models.